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CIRCULAR ANTIPODAL GRAPHS

J. VEERANJANEUYULU, P. L. N. VARMA

Abstract. In this article we introduce the concept of circular antipodal graphs. We study some properties as well as self-circular antipodal graphs. We construct circular antipodal graphs of some families of graphs. Also some results are obtained in case of trees.

Key words and phrases: circular distance, circular antipodal graphs, self-circular antipodal graphs.

1. Introduction

The concepts of antipodal vertices and antipodal graphs were introduced by D. H. Smith in 1971 (see [5]). Based on this concept many authors obtained results on antipodal graphs (for example see [1, 2, 3]). The antipodal graph of a graph H is defined by taking the same vertex set as H and two vertices are adjacent in antipodal graph if distance between them is equal to diameter of H (see[6]). A graph H is a self-antipodal graph, if it is isomorphic to antipodal graph of H.

The authors have introduced the concept of circular D-distance in [7]. Varma and Venkateswara Rao have introduced the concept of D-antipodal and detour D-antipodal graphs (see [8]). In the present work we introduce the concept of circular antipodal graphs based on circular distance.

2. Circular antipodal graphs

First we introduce the basic points of circular antipodal graphs.

Definition 2.1. Let H be a connected graph. A pair of vertices r,s are said to be *antipodal vertices* (respectively, *detour antipodal vertices*), distance between two vertices r, s is equal to the diameter of H, i.e., $d(r, s) = d(H)$ (respectively, $D(r, s) = D(H)$).

Definition 2.2. Let H be a graph. A vertex r of H is said to be *antipodal vertex* (respectively, *detour antipodal vertex*) of H, if there is another vertex t in H such that $d(r, t) = d(H)$ (respectively, $D(r, t) = D(H)$).

Now we define the circular antipodal graphs.

Definition 2.3. Let H be a connected graph. A pair of vertices r, s is said to be *circular antipodal vertices* if the circular distance between them is equal to the circular diameter of the graph. In otherwords r, s are *circular antipodal vertices* iff $cir(r, s) = d^C(H)$.

Definition 2.4. Let H be a graph. A vertex r of H is said to be *circular antipodal vertex* of H if there is another vertex t in H such that $cir(r, t) = d^C(H)$. The set of all circular antipodal vertices is called *circular antipodal vertex set* of H.

Definition 2.5. Let H be a graph. An edge $e = rs$ in H is said to be a *circular antipodal edge* of H, if the end vertices r, s are circular antipodal vertices. In other words any edge joining circular antipodal vertices is a circular antipodal edge.

Now we define circular antipodal graph.

Definition 2.6. Let H be a graph. The circular antipodal graph, $A^C(H)$ of H is defined as follows:
(1) vertex set of $A^C(H)$ is same as that of H.
(2) two vertices r, s are adjacent in $A^C(H)$ iff r, s are circular antipodal vertices in H.

Below theorem gives a class of graphs which are self-circular antipodal graphs.

Theorem 2.7. *The complete graph K_m is self-circular antipodal graph.*

Proof. Let K_m be a complete graph with m vertices. By definition, every pair of vertices is adjacent in K_m. Hence for any two vertices r, s in K_m, we have $cir(r, s) = m$. Also $d^C(K_m) = m$ (see theorem 3.1 in [9]). Therefore every pair of vertices is adjacent in circular antipodal graph. From this we can conclude that the circular antipodal graph of K_m is again K_m. Hence K_m is a self-circular antipodal graph.

3. Circular antipodal graphs of trees

In this section we study the circular antipodal graphs of trees. We prove that in a tree, only pendent vertices can be circular antipodal vertices.

Theorem 3.1. *For a tree T, the circular antipodal vertices are pendent vertices only. Non pendent vertices cannot be circular antipodal.*

Proof. Let T be a tree. The circular periphery $P^C(T)$ of T consists pendent vertices only. Let $r_i \in P^C(T)$ be a pendent vertex of T. Then there exists $r_j \in T$ such that $cir(r_i, r_j) = d^C(T)$. Because of symmetric property $cir(r_i, r_j) = cir(r_j, r_i) = d^C(T)$. Therefore both r_i, r_j are in circular periphery of T. Hence the circular antipodal vertices are pendent vertices only.

Non pendent vertices cannot be circular antipodal vertices. Suppose, in contrary, that r_i is not a pendent vertex. But r_i, r_j are circular antipodal vertices. From the definition of circular antipodal vertices, $cir(r_i, r_j) = d^C(T)$, i.e., r_i, r_j are in circular periphery of T, which is contradiction. Hence non pendent vertices cannot be circular antipodal vertices.

***Remark* 3.2**. In a tree T, every pendent vertex need not be circular antipodal vertex.
For example see the figure 1.

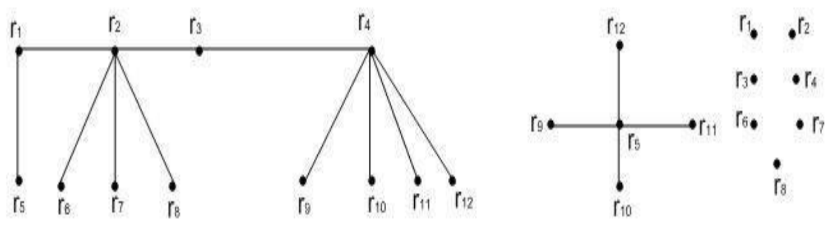

Figure 1. A Caterpillar and its circular antipodal graph

In this caterpillar the pendent vertices are $r_5, r_6, r_7, r_8, r_9, r_{10}, r_{11}, r_{12}$. But the circular antipodal vertices are only $r_5, r_9, r_{10}, r_{11}, r_{12}$. Hence it shows that a pendent vertex need not be circular antipodal vertex. Further H is a connected graph but circular antipodal graph of H is need not be connected.

Theorem 3.3. *For the path graph P_n, the circular antipodal graph consists of one copy of K_2 and n -2 isolated vertices.*

Proof. Let P_n be the path graph with vertex set $\{r_1,r_2,\cdots,r_n\}$. Then we can see that there is only one pair of vertices, namely, (r_1, r_n) such that $cir(r_1, r_n) = d^C(P_n)$. Therefore in the circular antipodal graph of P_n, only one pair is adjacent and all others are isolated vertices. Thus $A^C(P_n)$ consists of one copy of K_2 and n - 2 isolated vertices.

Theorem 3.4. *For the star graph $St_{1,n}$, circular antipodal graph consists of complete graph K_n and an isolated vertex.*

Proof. Let $St_{1,n}$ be a star graph. Let r_0 be the vertex which is adjacent to all other vertices $r_1,r_2,\cdots,r_n$. Then for any two distinct pendent vertices r_i, r_j, the circular distance between them is same as circular diameter, i.e., $cir(r_i,r_j) = d^C(St_{1,n})$ (see theorem 5.6 in [9]). The vertex r_0 is not an circular antipodal vertex of any other vertex. Therefore in circular antipodal graph of $St_{1,n}$, r_i is adjacent to all other end vertices. Hence the circular antipodal graph consists of one isolated vertex which is corresponding to r_0 and a complete graph, K_n, on n vertices.

4. Circular antipodal graphs of families of graphs

In this section, we construct circular antipodal graphs of some classes of graphs.

Theorem 4.1. *For a cyclic graph C_n, the circular antipodal graph consists of a copy of K_n.*

Proof. Let C_n be a cyclic graph with vertex set $\{r_1,r_2,\cdots,r_n\}$. Then we can see that, for any two vertices (r_i,r_j) we have $cir(r_i,r_j) = d^C(C_n)$ (see theorem 3.2 in [9]). Therefore in the circular antipodal graph of C_n, there exists an edge between every pair of vertices. Thus $A^C(C_n)$ consists of one copy of K_n.

Theorem 4.2. *For a wheel graph $W_{1,n}(n \geq 5)$ with $n+1$ vertices, the circular antipodal graph of $W_{1,n}$ consists of a n - 3 regular graph on n vertices and an isolated vertex.*

Proof. Let $W_{1,n}$ be a wheel graph with n+1 vertices. Let r_0 is adjacent to all other vertices $r_1,r_2,\cdots,r_n$. Then we have $cir(r_i,r_j) = d^C(W_{1,n}) = n+2$ (see theorem 3.3 in [9]), where j 6= $0,i-1,i+1$ for $i = 1,2,\cdots,n$ (for r_1 these vertices are r_0,r_2,r_n and for r_n these vertices are r_0,r_1,r_{n-1}). Thus for every r_i these are $n-3$ circular antipodal vertices. Hence the circular antipodal graph consists of a $n-3$ regular graph on n vertices and an isolated vertex.

Next, we consider the circular antipodal graph of a complete bipartite graph. We begin with a notation.

Notation 4.3. *In $K_{m,n}$, the vertex set is given by $V \cup W$, where V consists of m vertices $\{r_1,r_2,\cdots,r_m\}$ and W consists of n vertices $\{s_1,s_2,\cdots,s_n\}$.*

Theorem 4.4. *The circular antipodal graph of a complete bipartite graph $K_{m,n}(m<n)$ consists of the complete graph K_n on n vertices and m isolated vertices.*

Proof. In $K_{m,n}(m<n)$, we have $cir(r_i, r_j) = 2m < d^C(K_{m,n})$, $cir(s_i, s_j) = 2m+2 = d^C(K_{m,n})$ and $cir(r_i,s_j) = 2m < d^C(K_{m,n})$ (see theorem 3.4 in [9]). Therefore any pair s_i,s_j is adjacent in circular antipodal graph and no other pair is adjacent. Hence the circular antipodal graph $K_{m,n}$ consists of the complete graph, K_n, on n vertices and all other m vertices are isolated.

Theorem 4.5. *The circular antipodal graph of a complete bipartite graph $K_{m,m}$ consists of one copy of the complete graph K_{2m} on $2m$ vertices.*

Proof. In $K_{m,m}$, we have $cir(r_i,r_j) = 2m = d^C(K_{m,m})$, $cir(s_i,s_j) = 2m = d^C(K_{m,m})$ and $cir(r_i,s_j) = 2m = d^C(K_{m,m})$ (see theorem 3.4 in [9]). Therefore any pair is adjacent in the circular antipodal graph. Hence circular antipodal graph $K_{m,m}$ consists of the complete graph, K_{2m}, on $2m$ vertices.

5. Conclusion

The study of antipodal graphs has gained momentum in recent years. In this paper we study the circular antipodal graphs of some classes of graphs. Further study will be taken up about circular *D*-antipodal graphs.

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Department of Mathematics and Statistics,
School of Applied Science and Humanities,
Vignan's Foundation For Science, Technology & Research, Vadlamudi - 522213, AP, India.
***E-mail address*: jangam.vfstr@gmail.com, plnvarma@gmail.com**